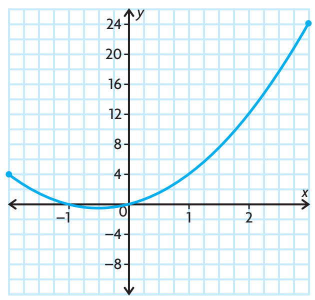
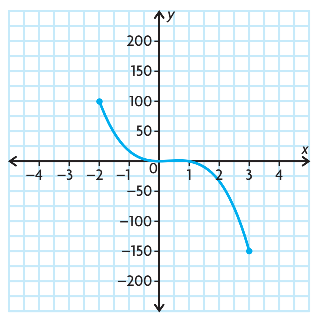
d)



e)

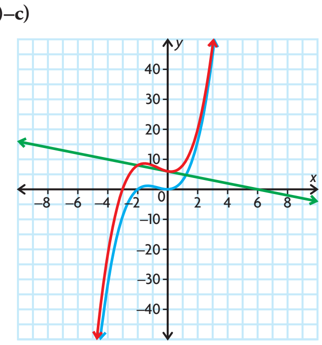


20. **a)** D
b) C
c) A
d) B

21. **a)**

x	−3	−2	−1	0	1	2
***f*(*x*)**	−9	0	1	0	3	16
***g*(*x*)**	9	8	7	6	5	4
(*f* + *g*)(*x*)	0	8	8	6	8	20

b)–c)

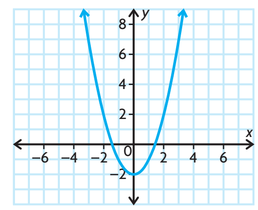


d) $x^3 + 2x^2 - x + 6$
e) Answers may vary. For example, (0, 0) belongs to *f*, (0, 6) belongs to *g* and (0, 6) belongs to $f + g$. Also, (1, 3) belongs to *f*, (1, 5) belongs to *g* and (1, 8) belongs to $f + g$.

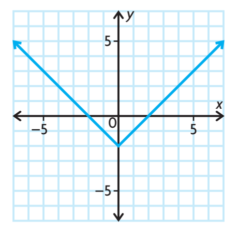
Chapter Self-Test, p. 62

1. **a)** Yes. It passes the vertical line test.
b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} | y \geq 0\}$

2. **a)** $f(x) = x^2$ or $f(x) = |x|$
b)



or

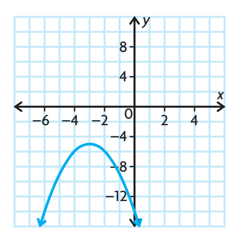


c) The graph was translated 2 units down.

3. $f(-x) = |3(-x)| + (-x)^2$
$= |3x| + x^2 = f(x)$

4. 2^x has a horizontal asymptote while x^2 does not. The range of 2^x is $\{y \in \mathbf{R} \mid y > 0\}$ while the range of x^2 is $\{y \in \mathbf{R} \mid y \geq 0\}$. 2^x is increasing on the whole real line and x^2 has an interval of decrease and an interval of increase.

5. reflection over the *x*-axis, translation down 5 units, translation left 3 units



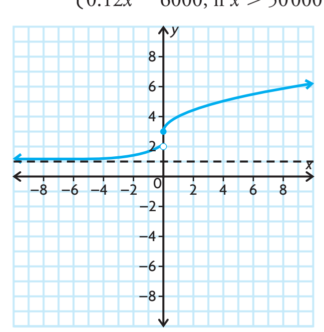
6. horizontal stretch by a factor of 2, translation 1 unit up;
$f(x) = \text{if} \left|\frac{1}{2}x\right| + 1$

7. **a)** $(-4, 17)$
b) $(5, 3)$

8. $f^{1}(x) = -\frac{x}{2} - 1$

9. **a)** \$9000
b) $f(x) = \begin{cases} 0.05, \text{ if } x \leq 50\,000 \\ 0.12x - 6000, \text{ if } x > 50\,000 \end{cases}$

10. **a)**



b) $f(x)$ is discontinuous at $x = 0$ because the two pieces do not have the same value when $x = 0$. When $x = 0$, $2^x + 1 = 2$ and $\sqrt{x + 3} = 3$.
c) Intervals of increase: $(-\infty, 0)$, $(0, \infty)$; no intervals of decrease
d) $D = \{x \in \mathbf{R}\}$,
$R = \{y \in \mathbf{R} \mid 0 < y < 2 \text{ or } y \geq 3\}$

Chapter 2

Getting Started, p. 66

1. **a)** $\frac{4}{3}$ **b)** $-\frac{6}{7}$

2. **a)** Each successive first difference is 2 times the previous first difference. The function is exponential.
b) The second differences are all 6. The function is quadratic.

3. **a)** $-\frac{3}{2}, 2$ **c)** 45°, 225°
b) 0 **d)** −270°, −90°

4. **a)** vertical compression by a factor of $\frac{1}{2}$
b) vertical stretch by a factor of 2, horizontal translation 4 units to the right
c) vertical stretch by a factor of 3, reflection across *x*-axis, vertical translation 7 units up
d) vertical stretch by a factor of 5, horizontal translation 3 units to the right, vertical translation 2 units down,

5. **a)** $A = 1000(1.08)^t$
b) \$1259.71
c) No, since the interest is compounded each year, each year you earn more interest than the previous year.

6. **a)** 15 m; 1 m
b) 24 s
c) 15 m

7.

Linear relations	**Nonlinear relations**
constant; same as slope of line; positive for lines that slope up from left to right; negative for lines that slope down from left to right; 0 for horizontal lines.	variable; can be positive, negative, or 0 for different parts of the same relation

Rates of Change

Lesson 2.1, pp. 76–78

1. **a)** 19 **c)** 13 **e)** 11.4
b) 15 **d)** 12 **f)** 11.04

2. **a)** **i)** 15 m/s **ii)** −5 m/s

b) During the first interval, the height is increasing at 15 m/s; during the second interval, the height is decreasing at 5 m/s.

3. $f(x)$ is always increasing at a constant rate. $g(x)$ is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$, so the rate of change is not constant.

4. **a)** 352, 138, 286, 28, 60, -34 people/h
b) the rate of growth of the crowd at the rally
c) A positive rate of growth indicates that people were arriving at the rally. A negative rate of growth indicates that people were leaving the rally.

5. **a)** 203, 193, 165, 178.5, 218.5, 146 km/day
b) No. Some days the distance travelled was greater than others.

6. 4; 4; the average rate of change is always 4 because the function is linear, with a slope of 4.

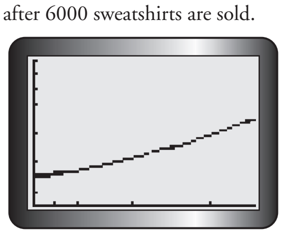
7. The rate of change is 0 for 0 to 250 min. After 250 min, the rate of change is \$0.10/min.

8. **a)** **i)** 750 people/year
ii) 3000 people/year
iii) 12 000 people/year
iv) 5250 people/year
b) No; the rate of growth increases as the time increases.
c) You must assume that the growth continues to follow this pattern, and that the population will be 5 120 000 people in 2050.

9. -2 m/s

10. **a)** **i)** \$2.60/sweatshirt
ii) \$2.00/sweatshirt
iii) \$1.40/sweatshirt
iv) \$0.80/sweatshirt
b) The rate of change is still positive, but it is decreasing. This means that the profit is still increasing, but at a decreasing rate.
c) No; after 6000 sweatshirts are sold, the rate of change becomes negative. This means that the profit begins to decrease after 6000 sweatshirts are sold.

11. **a)**



b) The rate of change will be greater farther in the future. The graph is getting steeper as the values of t increase.

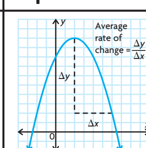
c) **i)** 1500 people/year
ii) 1700 people/year
iii) 2000 people/year
iv) 2500 people/year
d) The prediction was correct.

12. Answers may vary. For example:
a) Someone might calculate the average increase in the price of gasoline over time. One might also calculate the average decrease in the price of computers over time.
b) An average rate of change might be useful for predicting the behaviour of a relationship in the future.
c) An average rate of change is calculated by dividing the change in the dependent variable by the corresponding change in the dependent variable.

13. -7.8%

14. Answers may vary. For example:

AVERAGE RATE OF CHANGE

Definition in your own words	Personal example	Visual representation
the change in one quantity divided by the change in a related quantity	I record the number of miles I run each week versus the week number. Then, I can calculate the average rate of change in the distance I run over the course of weeks.	

15. 80 km/h

Lesson 2.2, pp. 85–88

1. **a)**

Preceding Interval	$\Delta f(x)$	Δx	Average Rate of Change, $\frac{\Delta f(x)}{\Delta x}$
$1 \leq x \leq 2$	$13 - (-2) = 15$	$2 - 1 = 1$	15
$1.5 \leq x \leq 2$	8.75	0.5	17.5
$1.9 \leq x \leq 2$	1.95	0.1	19.5
$1.99 \leq x \leq 2$	0.1995	0.01	19.95

Following Interval	$\Delta f(x)$	Δx	Average Rate of Change, $\frac{\Delta f(x)}{\Delta x}$
$2 \leq x \leq 3$	$38 - 13 = 25$	$3 - 2 = 1$	25
$2 \leq x \leq 2.5$	11.25	0.5	22.5
$2 \leq x \leq 2.1$	2.05	0.1	20.5
$2 \leq x \leq 2.01$	0.2005	0.01	20.05

b) 20

2. **a)** 5.4 m/s **b)** 5.4 m/s
c) Answers may vary. For example: I prefer the centred interval method. Fewer calculations are required, and it takes into account points on each side of the given point in each calculation.

3. **a)** 200
b) 40 raccoons/month
c) 50 raccoons/month
d) The three answers represent different things: the population at a particular time, the average rate of change prior to that time, and the instantaneous rate of change at that time.

4. **a)** -24 **b)** 0 **c)** 48 **d)** 96

5. -27 m/s

6. \$11 610 per year

7. **a)** 0 people/year
b) Answers may vary. For example: Yes, it makes sense. It means that the populations in 2000 and 2024 are the same, so their average rate of change is 0.
c) The average rate of change from 2000 to 2012 is 18 000 people/year; the average rate of change from 2012 to 2024 is $-18\,000$ people/year.
d) $t = 12$

8. About $-\$960$ per year; when the car turns five, it loses \$960 of its value.

9. **a)** 1.65 s **b)** about 14 m/s

10. 100π cm^3/cm

11. If David knows how far he has travelled and how long he has been driving, he can calculate his average speed from the beginning of the trip by dividing the distance travelled by the time he has been driving.

12. **a)** -22.5 °F/min
b) Answers may vary. For example: -25.5 °F/min
c) Answers may vary. For example, the first rate is using a larger interval to estimate the instantaneous rate.
d) Answers may vary. For example, the second estimate is better, as it uses a much smaller interval to estimate the instantaneous rate.

13. Answers may vary. For example:

Method of Estimating Instantaneous Rate of Change	Advantage	Disadvantage
series of preceding intervals and following intervals	accounts for differences in the way that change occurs on either side of the given point	must do two sets of calculations
series of centred intervals	accounts for points on either side of the given interval in same calculation	to get a precise answer, numbers involved will need to have several decimal places
difference quotient	more precise	calculations can be tedious or messy

14. **a)** $100\pi\ cm^2/cm$
b) $240\pi\ cm^2/cm$

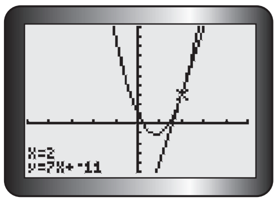
15. $36\ cm^2/cm$

16. $160\pi\ cm^2/cm$

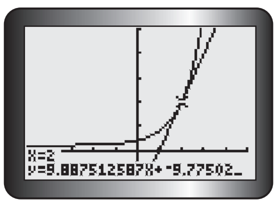
Lesson 2.3, pp. 91–92

1. **a)** about 7 **c)** about 0.25
b) about 10 **d)** 2

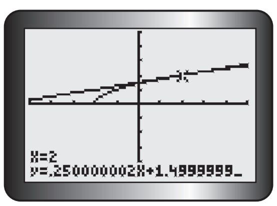
2. **a)**



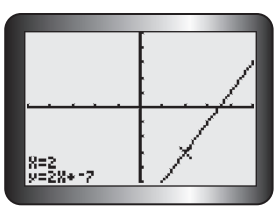
b)



c)

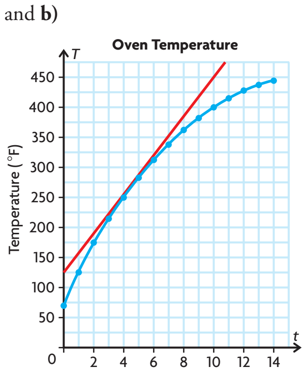


d)



3. **a)** Set A: 0, 0, 0, 0
Set B: 14, 1.4, 5, 0.009
Set C: -4, -0.69, -3, -0.009
b) Set A: All slopes are zero.
Set B: All slopes are positive.
Set C: All slopes are negative.

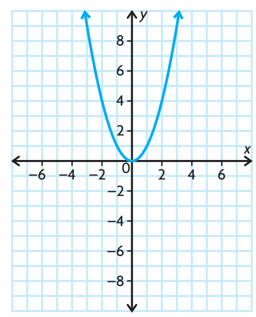
4. **a)** and **b)**



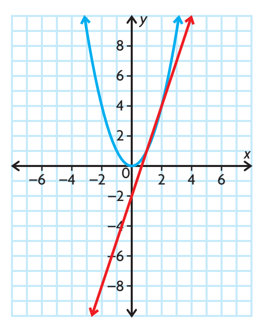
c) 31
d) Rate of change is about 30 °F/min at $x = 5$.
e) Answers may vary. For example: The two answers are about the same. The slope of the tangent line at the point is the same as the instantaneous rate of change at the point.

5. Answers may vary. For example: Similarity: the calculation; difference: average rate of change is over an interval; instantaneous rate of change is at a point.

6. **a)**



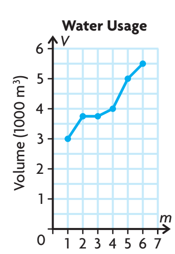
b)



c) (1.5, 2.25)

Mid-Chapter Review, p. 95

1. **a)**



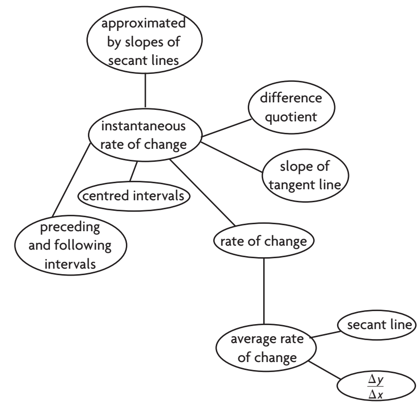
b) 750; 250; 1100; 400 m^3/month
c) April and May
d) 580 m^3/month

2. **a)** The equation models exponential growth. This means that the average rate of change between consecutive years will always increase.
b) The instantaneous rate of change in population in 2010 is about 950 people per year.

3. **a)** 10 m/s; -10 m/s
b) $t = 2$; Answers may vary. For example: The graph has a vertex at (2, 21). It appears that a tangent line at this point would be horizontal.
$\frac{(f(2.01) - f(1.99))}{0.02}$

4. 0.9 m/day

5. Answers may vary. For example:



6. Answers may vary. For example:

Points	Slope of Secant
(2, 9) and (1, 2)	7
(2, 9) and (1.5, 4.375)	9.25
(2, 9) and (1.9, 7.859)	11.41
(2, 9) and (2.1, 10.261)	12.61
(2, 9) and (2.5, 16.625)	15.25
(2, 9) and (3, 28)	19

The slope of the tangent line at (2, 9) is about 12.

7. 4

8. The instantaneous rate of change of the function whose graph is shown is 4 at $x = 2$.

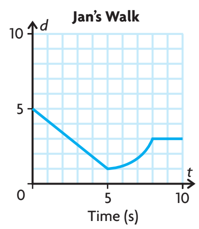
9. Answers may vary. For example:
a) 0 **b)** 4 **c)** 5 **d)** 8

Lesson 2.4, pp. 103–106

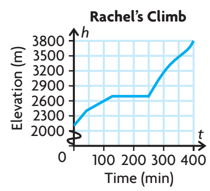
1. **a)** C **b)** A **c)** B

2. All of the graphs show that the speed is constant. In a), the speed is positive and constant. In b), the speed is negative and constant. In c), the speed is 0, which is constant.

3.

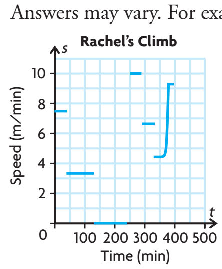


4. **a)** Answers may vary. For example:

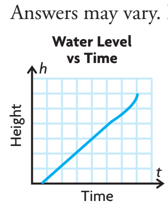


b) Average speed over first 40 min is 7.5 m/min, average speed over next 90 min is 3.3 m/min, average speed over next 120 min is 0 m/min, average speed over next 40 min is 10 m/min, average speed over next 45 min is 6.7 m/min, and average speed over last 60 min is 5.7 m/min.

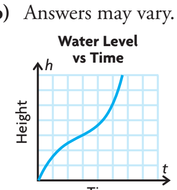
c) Answers may vary. For example:



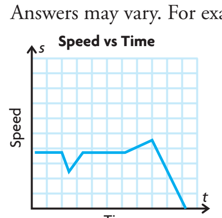
5. **a)** Answers may vary. For example:



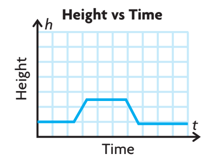
b) Answers may vary. For example:



6. **a)** Answers may vary. For example:



b) Answers may vary. For example:

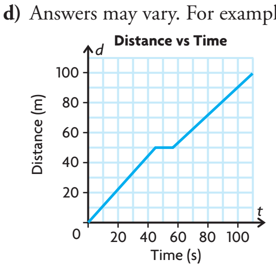


7. **a)** 1.11 m/s

b) 0.91 m/s

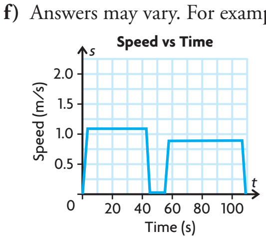
c) The graph of the first length would be steeper, indicating a quicker speed. The graph of the second length would be less steep, indicating a slower speed.

d) Answers may vary. For example:



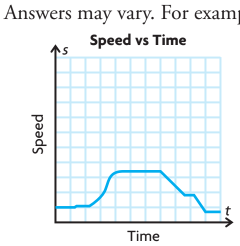
e) 0 m/s

f) Answers may vary. For example:



8. **a)** A **b)** C **c)** D **d)** B

9. Answers may vary. For example:

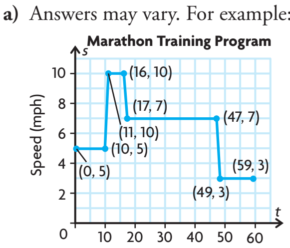


10. **a)** and **b)**

i) Start 5 m from sensor. Walk toward sensor at a constant rate of 1 m/s for 3 s. Walk away from sensor at a constant rate of 1 m/s for 3 s.

ii) Start 6 m from sensor. Walk toward sensor at a constant rate of 1 m/s for 2 s. Stand still for 1 s. Walk toward sensor at a constant rate of 1 m/s for 2 s. Walk away from sensor at a constant rate of 1.5 m/s.

11. **a)** Answers may vary. For example:



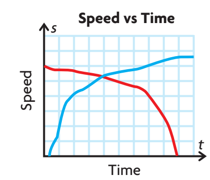
b) 5 mph/min

c) −0.1842 mph/min

d) The answer to part c) is an average rate of change over a long period, but the runner does not slow down at a constant rate during this period.

12. Answers may vary. For example: Walk from (0, 0) to (5, 5) and stop for 5 s. Then run to (15, 30). Continue walking to (20, 5) and end at (25, 0). What is the maximum speed and minimum speed on an interval? Create the speed versus time graph from these data.

13. Answers may vary. For example:



14. If the original graph showed an increase in rate, it would mean that the distance travelled during each successive unit of time would be greater—meaning a graph that curves upward. If the original graph showed a straight, horizontal line, then it would mean that the distance travelled during each successive unit of time would be greater—meaning a steady increasing straight line on the second graph. If the original graph showed a decrease in rate, it would mean that the distance travelled during each successive unit of time would be less—meaning a line that curves down.

Lesson 2.5, pp. 111–113

1. Answers may vary. For example, I used the difference quotient when $a = 1.5$ and $h = 0.001$ and got an estimate for the instantaneous rate of change in cost that was close to 0.

2. 0

3. **a)** The slopes of the tangent lines are positive, but close to 0.

b) The slopes of the tangent lines are negative, but close to 0.

4. **a)** The slopes of the tangent lines are negative, but close to 0.

b) The slopes of the tangent lines are positive, but close to 0.

5. **a)** The slope is 0.

b) The slope is 0.

c) The slope is 0.

d) The slope is 0.

6. **a)** minimum

b) maximum

c) minimum

d) maximum

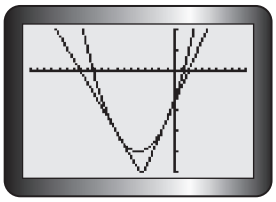
e) maximum

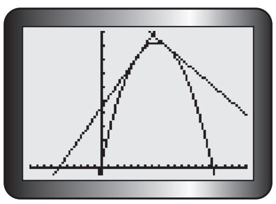
f) maximum

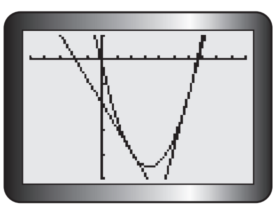
Answers

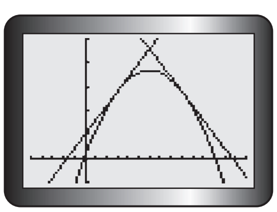
7. $t = 2.75$; Answers may vary. For example: The slopes of tangents for values of t less than about 2.75 would be positive, while slopes of tangents for values of t greater than about 2.75 would be negative.

8. a) $x = -5$; minimum
$x = 7.5$; maximum
$x = 3.25$; minimum
$x = 6$; maximum

b) i)


ii)


iii)


iv)


c) Answers may vary. For example, if the sign of the slope of the tangent changed from positive to negative, there was a maximum. If the sign of the slope of the tangent changed from negative to positive, there was a minimum.

9. a) i) maximum = (0, 100); minimum = (5, 44.4)
ii) maximum = (10, 141.6); minimum = (0, 35)

b) For an equation that represents exponential growth (where $r > 0$), the minimum value will always be at point a and the maximum value will always be at point b, because y will always increase as x increases. For an equation that represents exponential decay (where $r < 0$), the minimum value will always be at point b and the maximum value will always be at point a, because y will always decrease as x increases.

10. Answers may vary. For example, the slope of the tangent at 0.5 s is 0. The slope of the tangent at 0 s is 5, and the slope of the tangent at 1 s is -5. So, the diver reaches her maximum height at 0.5 s.

11. Answers may vary. For example, yes, this observation is correct. The slope of the tangent at 1.5 s is 0. The slopes of the tangents between 1 s and 1.5 s are negative, and the slopes of the tangent lines between 1.5 s and 2 s are positive. So, the minimum of the function occurs at 1.5 s.

12. Answers may vary. For example, estimate the slope of the tangent line to the curve when $x = 5$ by writing an equation for the slope of any secant line on the graph of $R(x)$. If the slope of the tangent is 0, this will confirm there may be a maximum at $x = 5$. If the slopes of tangent lines to the left are positive and the slopes of tangent lines to the right are negative, this will confirm that a maximum occurs at $x = 5$.

13. Answers may vary. For example, because $\sin 90°$ gives a maximum value of 1, I know that a maximum occurs when $(k(x - d)) = 90°$. Solving this equation for x will tell me what types of x-values will give a maximum. For example, when $k = 2$ and $d = 3$,
$(2(x - 3°)) = 90°$
$(x - 3°) = 45°$
$x = 48°$

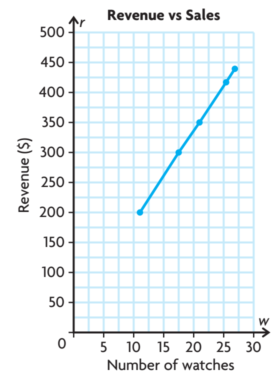
14. Myra is plotting (instantaneous) velocity versus time. The rates of change Myra calculates represent acceleration. When Myra's graph is increasing, the car is accelerating. When Myra's graph is decreasing, the car is decelerating. When Myra's graph is constant, the velocity of the car is constant; the car is neither accelerating nor decelerating.

15. $-4, -2, 4, 6$; The rule appears to be "multiply the x-coordinate by 2." 12, 3, 12, 27; The rule for $f(x) = x^3$ seems to be "square the x-coordinate and multiply by 3."

Chapter Review, pp. 116–117

1. a) Yes. Divide revenue by number of watches, and the slope is 17.5.

b) Answers may vary. For example:



The data represent a linear relationship.

c) $17.50 per watch

d) $17.50; this is the slope of the line on the graph.

2. a) 1.5 m/s
b) -1.5 m/s
c) The time intervals have the same length. The amount of change is the same, but with opposite signs for the two intervals. So, the rates of change are the same for the two intervals, but with opposite signs.

3. a) $E = 2500m + 10\,000$
b) $2500 per month
c) No; the equation that represents this situation is linear, and the rate of change over time for a linear equation is constant.

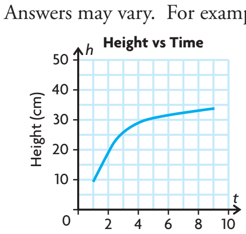
4. a) Answers may vary. For example, because the unit of the equation is years, you would not choose $3 \le t \le 4.25$ and $4 \le t \le 5$. A better choice would be $3.75 \le t \le 4.0$ and $4.0 \le t \le 4.25$.
b) Answers may vary. For example, find the average of the two interval values: $\frac{(600.56 + 621.91)}{2} = \$611.24$

5. a) Answers may vary. For example, squeezing the interval.
b) 4.19 cm/s

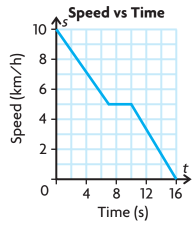
6. a) -2 b) 0 c) 4

7. a) -37 b) -17 c) 0 d) 23

8. Answers may vary. For example:



9. a) Answers may vary. For example:

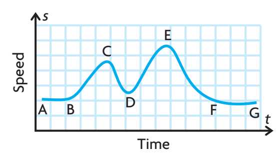


b) $-\frac{5}{7}$ km/h/s

c) From (7, 5) to (12, $\frac{10}{3}$), the rate of change of speed in $-\frac{1}{3}$ km/h/s

d) $-\frac{5}{6}$ km/h/s

10. The roller coaster moves at a slow steady speed between A and B. At B, it begins to accelerate as it moves down to C. Going uphill from C to D it decelerates. At D, it starts to move down and accelerates to E, where the speed starts to decrease until F, where it maintains a slower speed to G, the end of the track.

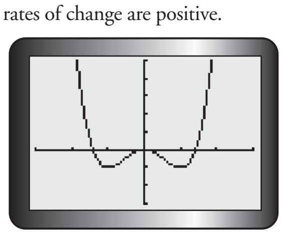


11. a) minimum d) minimum
b) maximum e) minimum
c) maximum f) maximum

12. a) i) $m = h - 26$ ii) $m = -4h - 48$
b) i) $m = -26$ ii) $m = -48$

13. a) To the left of a maximum, the instantaneous rates of change are positive. To the right, the instantaneous rates of change are negative.
b) To the left of a minimum, the instantaneous rates of change are negative. To the right, the instantaneous rates of change are positive.

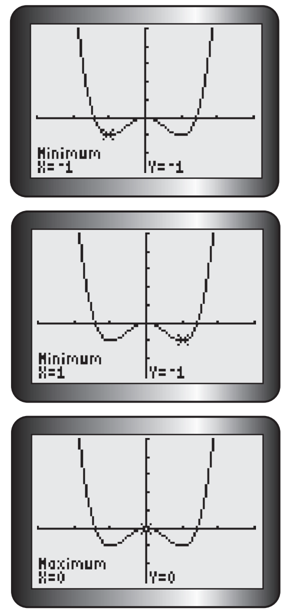
14. a)



b) minimum: $x = -1$, $x = 1$
maximum: $x = 0$

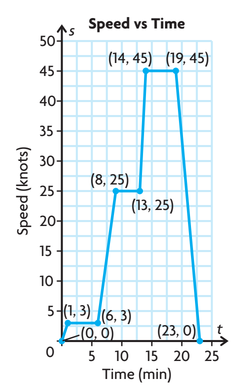
c) The slopes of tangent lines for points to the left of a minimum will be negative, while the slopes of tangent lines for points to the right of a minimum will be positive. The slopes of tangent lines for points to the left of a maximum will be positive, while the slopes of tangent lines for points to the right of a minimum will be negative.

d)



Chapter Self-Test, p. 118

1. a)



b) 11 kn/min; 0 kn/min; the two different average rates of change indicate that the boat was increasing its speed from $t = 6$ to $t = 8$ at a rate of 11knots/min and moving at a constant speed from $t = 8$ to $t = 13$.

c) 11 kn/min

2. a) -1
b) The hot cocoa is cooling by 1 °C/min on average.
c) -0.75
d) The hot cocoa is cooling by 0.75 °C/min after 30 min.
e) The rate decreases over the interval, until it is nearly 0 and constant.

3. a) \$310 per dollar spent
b) $-$\$100 per dollar spent
c) The positive sign for part a) means that the company is increasing its profit when it spends between \$8000 and \$10 000 on advertising. The negative sign means the company's profit is decreasing when it spends \$50 000 on advertising.

4. a) -1; 0 (minimum); 7
b) 4.5; -4.5; 0 (maximum)

Chapter 3

Getting Started, p. 122

1. a) $6x^3 - 22x^2$
b) $x^2 + 2x - 24$
c) $24x^3 - 44x^2 - 40x$
d) $5x^3 + 31x^2 - 68x + 32$

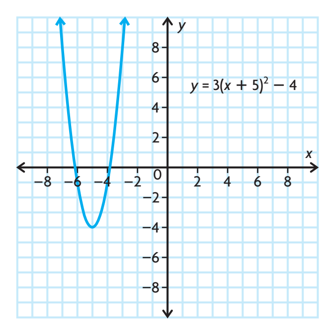
2. a) $(x + 7)(x - 4)$
b) $2(x - 2)(x - 7)$

3. a) $x = -6$
b) $x = -3, 4.5$
c) $x = -3, -8$
d) $x = \frac{1}{3}, -4$

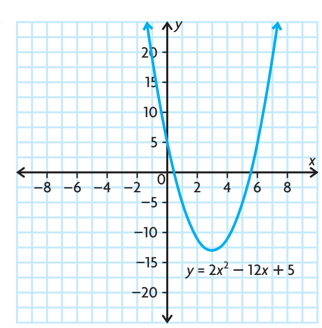
4. a) vertical compression by a factor of $\frac{1}{4}$; horizontal translation 3 units to the right; vertical translation 9 units up
b) vertical compression by a factor of $\frac{1}{4}$; vertical translation 7 units down

5. a) $y = 2(x - 5)^2 - 2$
b) $y = -2x^2 + 3$

6. a)



b)



7. a) quadratic
b) other
c) other
d) linear